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Association between basic numerical abilities and mathematics achievement

Delphine Sasanguie[1,2*], Bert De Smedt[3], Emmy Defever[1,2] and Bert Reynvoet[1,2]

[1]Department of Psychology, University of Leuven, Belgium
[2]Subfaculty of Psychology and Educational Sciences, University of Leuven, Campus Kulak, Leuven, Belgium
[3]Parenting and Special Education Research Group, University of Leuven, Belgium

Various measures have been used to investigate number processing in children, including a number comparison or a number line estimation task. The present study aimed to examine whether and to which extent these different measures of number representation are related to performance on a curriculum-based standardized mathematics achievement test in kindergarteners, first, second, and sixth graders. Children completed a number comparison task and a number line estimation task with a balanced set of symbolic (Arabic digits) and non-symbolic (dot patterns) stimuli. Associations with mathematics achievement were observed for the symbolic measures. Although the association with number line estimation was consistent over grades, the association with number comparison was much stronger in kindergarten compared to the other grades. The current data indicate that a good knowledge of the numerical meaning of Arabic digits is important for children's mathematical development and that particularly the access to the numerical meaning of symbolic digits rather than the representation of number *per se* is important.

Infants have an awareness of numerical magnitude and are able to represent quantities (Feigenson, Dehaene, & Spelke, 2004; Piazza, 2010). This understanding of quantity develops over time and allows even very young children to compare, estimate, or add numerical magnitudes (Xu, Spelke, & Goddard, 2005). By repeatedly linking a quantity with its associated numeral, children acquire a symbolic system to represent number, that is mapped onto a pre-existing, approximate non-symbolic number system, or ANS (Barth, La Mont, Lipton, & Spelke, 2005; Mundy & Gilmore, 2009; but see Carey, 2004 for an alternative interpretation). The ability to represent number is proposed to be a key precursor of later mathematical development. De Smedt, Verschaffel, and Ghesquière (2009) provided the first longitudinal evidence that the speed of comparing

*Correspondence should be addressed to Delphine Sasanguie, Department of Psychology, Subfaculty of Psychology and Educational Sciences, K.U.Leuven – Campus Kulak, Etienne Sabbelaan 53; 8500 Kortrijk, Belgium (e-mail: Delphine.Sasanguie@kuleuven-kortrijk.be).

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numbers assessed at the start of formal schooling is predictively related to individual differences in mathematics achievement in second grade. Furthermore, children with mathematical disabilities seem to have particular deficits in understanding number (De Smedt, Reynvoet *et al.*, 2009; Geary, Hoard, Nugent, & Byrd-Craven, 2008; Piazza *et al.*, 2010; Rouselle & Noël, 2007).

Two accounts for these impairments in number representation have been put forward. These impairments might arise from a deficit in the access to magnitude from symbols, known as the *access deficit hypothesis* (De Smedt & Gilmore, 2011; Landerl & Kölle, 2009; Rouselle & Noël, 2007). On the other hand, these impairments might be due to deficits in the internal magnitude representation, as proposed by the *defective number module hypothesis* (Butterworth, 2005; Mussolin, Mejias, & Noel, 2010).

In typically developing children, measures of number processing, such as number comparison (Bugden & Ansari, 2011; Holloway & Ansari, 2009; De Smedt, Verschaffel *et al.*, 2009; Landerl & Kölle, 2009) and number line estimation (Booth & Siegler, 2006, 2008; Siegler & Booth, 2004), have been related to individual differences in mathematical achievement. Most of the existing studies have largely focused on symbolic representations and research that investigates both symbolic and non-symbolic stimuli in one sample is scarce. Moreover, these studies typically focused on one age group rather than investigating subjects from various ages. The present study therefore extends the existing body of evidence by (1) examining associations with symbolic and non-symbolic tasks (2) investigating these associations at various ages (from kindergarten to sixth grade).

Two types of tasks have been used to investigate number processing, that is, number comparison and number line estimation. In number comparison, participants have to indicate which of two numbers is the larger. Both the accuracy (e.g., Piazza *et al.*, 2010; Soltész, Szücs, & Szücs, 2010) and speed (De Smedt, Verschaffel *et al.*, 2009; Holloway & Ansari, 2009; Landerl & Kölle, 2009; Mundy & Gilmore, 2009) are correlated with individual differences in mathematical achievement. During this task, a comparison distance effect (CDE) is typically observed: children are slower and less accurate in deciding which of two numbers is the larger when these numbers are numerically close to each other (e.g., 8 vs. 9) relative to comparisons of numbers that are further away (e.g., 1 vs. 9). This effect is explained by assuming that number are represented mentally akin to a left-to-right oriented *mental number line* as a Gaussian distribution around the true location of each specific number, with partially overlapping representations of other nearby numbers (Dehaene, 1997). Whenever a number is presented, its representation and the representation of neighbouring numbers will be partly activated, resulting in more difficult discriminations for nearby numbers. Sekuler and Mierkiewicz (1977) demonstrated that the CDE is already observed in 5-year-olds, with the size of this effect decreasing with increasing age. The size of the CDE on symbolic but not non-symbolic comparison tasks correlates with mathematical achievement, with larger CDEs being associated with poorer mathematical achievement (De Smedt, Verschaffel *et al.*, 2009; Holloway & Ansari, 2009; Soltész *et al.*, 2010; but see Schneider, Grabner, & Paetsch, 2009), a finding that is consistent with data in children with mathematical disabilities where deficits on symbolic but not non-symbolic tasks are observed (e.g., De Smedt & Gilmore, 2011; Landerl & Kölle, 2009; Rouselle & Noel, 2007). However, Halberda, Mazzocco, & Feigenson (2008) found that the acuity of the ANS, determined by the Weber fraction on a non-symbolic comparison task at the age of 14 correlates with retrospective measures of mathematical achievement from 5 years on. Although, it has to be stressed that in this study, instead of the CDE, the Weber fraction was applied as a measure of numerical processing. The Weber fraction measures the smallest numerical change to a stimulus that can be reliably detected (Halberda & Feigenson, 2008). The Weber fraction is based on accuracies, whereas the CDE is computed as a difference between conditions based on reaction times (RTs).

In the number line estimation, participants have to put a number on an empty number line from, for example, 0 to 10, 0 to 100, or 0 to 1,000 (e.g., Booth & Siegler, 2006, 2008; Siegler & Booth, 2004; Siegler & Ramani, 2009). Over development, the estimation pattern of children makes a shift from a logarithmic to a linear representation (Berteletti, Lucangeli, Piazza, Dehaene, & Zorzi, 2010; Opfer & Siegler, 2007; Siegler & Booth, 2004) and the linearity of these representations is associated with mathematical achievement in primary school (Booth & Siegler, 2006; Schneider *et al*., 2009; Siegler & Booth, 2004). Data in children with mathematical disabilities are in line with this, indicating that these children are less accurate in numerical estimation and rely more on a logarithmic representations (Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007). Each of the aforementioned studies assessed number line estimation with a symbolic task. Only a few studies have used non-symbolic number line estimation tasks (Barth, Starr, & Sullivan, 2009; Ebersbach, Luwel, Frick, Onghena, & Verschaffel, 2008), but there exists, to our knowledge, no study that examined the association between non-symbolic number line estimation and mathematics achievement.

The number comparison task and the number line estimation task differ in their characteristics and focus. Number line tasks are typically untimed and therefore measure the accuracy or preciseness of number representations. Number comparison tasks, on the other hand, are timed and are usually performed with very high accuracy, yet the speed on this these tasks reflects the fluency with which number representations are available. Few studies have administered a number comparison task and a number line task in one study and examined their relationship to mathematical ability (Laski & Siegler, 2007; Schneider *et al*., 2009). However, there are currently no studies that investigated both tasks and their relationship with mathematical achievement with both symbolic and non-symbolic stimuli.

The present study tried to address this issue by administering a number comparison task and number line estimation task in both symbolic and non-symbolic notations. Extending previous literature, we administered these tasks at various age points, that is, in kindergarteners, first, second, and sixth graders. Siegler and colleagues (Booth & Siegler, 2006; Siegler & Booth, 2004) already showed that first and second graders are the most interesting age groups to investigate, due to their logarithmic-to-linear shift at that age. Similar developmental trends are found at this age in number comparison (Holloway & Ansari, 2009). Kindergarteners were included, because this group of children did not get formal instruction and sixth graders were investigated to evaluate these associations in older children.

Against the background of the studies reviewed above, we expected an association between mathematics achievement and the symbolic number comparison task in young children and an association between mathematics achievement and both symbolic and non-symbolic number comparison in older children. We also hypothesized that the association between number comparison and mathematics achievement would decrease with age. We further expected that children who generated more linear patterns of estimates on the number line tasks would have higher mathematics achievement levels. With regard to the non-symbolic number line task, possible predictions were less clear. Against the background of the data by Halberda *et al*. (2008) on children's acuity of non-symbolic number representations, we expected an association between non-symbolic number line estimation and mathematics.

To investigate whether the association between the number processing tasks and mathematics achievement was specific, we also administered a curriculum-based spelling test. We also measured the knowledge of the Arabic digits 1–9 and the numerosities they represent with a number knowledge task in kindergartners and first graders to ensure that they were able to perform the administered numerical tasks.

Method

Participants

Participants were 127 typically developing children. Two children were excluded from the sample because they did not complete the mathematics achievement test. Seven children were removed because they responded too slowly or made too many errors (more than 3*SD* above the grade average). The final sample consisted of 118 subjects comprising 26 kindergartners (M_{age} = 5.6 years, 13 males), 30 first graders (M_{age} = 6.7 years, 11 males), 29 second graders (M_{age} = 7.6 years, 11 males), and 33 sixth graders (M_{age} = 11.6 years, 10 males).

Procedure

The curriculum-based standardized tests were administered at the beginning of February (about halfway the school year). The number processing tasks were administered 1 month later. The children were tested in a separate room. Kindergartners were tested in groups of 4 children and the other children in groups of 7–10 children. The number line estimation task (symbolic and non-symbolic condition) was assessed first after followed by the number comparison task (symbolic and non-symbolic condition). A short break was provided after each condition. The participants received a small reward.

Measures

Standardized tests

Mathematics. Mathematics achievement was assessed with a curriculum-based standardized achievement test for mathematics from the Flemish Student Monitoring System (Dudal, 2000a). This test consists of 60 items covering number knowledge, understanding of operations, (simple) arithmetic, word problem solving, measurement, and geometry. Cronbach's α for this test was .90, .92, and .80 for the first, second, and sixth grade, respectively. In kindergarten, mathematics achievement was assessed by the 'Numerical understanding, start last year of kindergarten' (Verachtert & Dudal, 2004). This test comprises 40 items covering magnitude comparison, understanding place value and space, ordinal numbers to 10, mathematics language and counting. Cronbach's α for this test was .93 (Verachtert, 2003).

Spelling. The curriculum-based standardized spelling test of the Flemish Student Monitoring System (Dudal, 2000b) was used to measure children's spelling skills. This test involved the dictation of letters, words, and sentences. Cronbach's α of this test was .94, .90, and .89 for the first, second, and sixth grade, respectively. In kindergarteners, we used a standardized language test (Bernaerts & Verniers, 2005), which consists of various phonological tasks, such as splitting a word into syllables, rhyming, listening to a short story and combining sound groups into one word. Cronbach's α for this test was .89.

Experimental tasks

Number knowledge task. Children had to read aloud the numbers 1–9 printed on the left side of the sheet. Next, they had to connect those digits to pictures on the right side of the sheet representing different numbers of clowns. Each participant was given one point for each correct association.

Number comparison task. The number comparison task was conducted using laptops with 14-inch screens. Stimulus presentation and the recording of behavioural data were controlled by E-prime 1.1 (Psychology Software Tools, http://www.pstnet.com). Participants had to select the larger of two presented quantities, one on the left and one on the right of the screen, by pressing a key at the side of the largest quantity. Children were asked to respond as quickly as possible without making errors. Five practice trials were included per task, to make children familiar with the task requirements. Stimuli involved all numbers between 1 and 9, but only combinations of stimuli with a maximum distance of 5 were presented to the children, which resulted in 60 trials per condition. A trial started with a fixation cross for 600 ms, after which the two stimuli that had to be compared appeared. Stimuli were presented simultaneously in the centre of the screen, 4.25 cm left and right, and remained on the screen until the child responded. The intertrial interval was 1,000 ms.

Symbolic stimuli involved Arabic digits (Arial font, 16) presented in white on a black background. Non-symbolic stimuli were two white-filled circles (radius 3.5 cm) each containing a set of black dots, were simultaneously presented on a black background. Dot patterns were generated with a MatLab script (Dehaene, Izard, and Piazza, 2005) and dot size and total area were systematically varied to prevent the consistent use of perceptual features to compare the dot arrays.

Number line estimation. Children were presented with 25 cm long lines in the centre of white A4 sheets. Two different intervals (0–10 and 0–100) were administered in both symbolic and non-symbolic notations. Symbolic stimuli were Arabic digits (Arial font size 16). Non-symbolic stimuli were white-filled circles containing a set of black dots, which were generated with the same MatLab script (Dehaene *et al.*, 2005) as in the number comparison task. The end points of the number lines were labelled on the left by 0 and on the right by either 10 or 100 in the symbolic condition and by an empty circle on the left and a circle with 10 or 100 dots on the right in the non-symbolic condition. The to be positioned quantity was shown in the centre of the sheet, 2 cm above the number line. All numbers and dot patterns except for 0 and 10 had to be positioned on the 0–10 interval, whereas for the 0–100 interval quantities were 2, 3, 4, 6, 18, 25, 48, 67, 71, and 86 (corresponding to sets A and B for the same interval used in Siegler & Opfer, 2003). The presentation order of the quantities was randomized and each line was presented on a separate sheet. Children were instructed to mark on the line were they thought that the quantity had to be positioned. To ensure that children were aware of the interval size, an example was provided by the experimenter solving the first item of the task

Table 1. Mean error rates, mean adjusted reaction times (RTs), and corresponding (standard deviations) for the five distances (d1–d5) of the symbolic condition of the number comparison task, per grade

	d1	d2	d3	d4	d5
Error rates (%)					
Kindergarteners	.19 (.14)	.16 (.12)	.13 (.11)	.15 (.15)	.07 (.10)
First graders	.11 (.08)	.04 (.07)	.03 (.04)	.02 (.04)	.02 (.04)
Second graders	.09 (.08)	.07 (.08)	.02 (.04)	.03 (.06)	.02 (.04)
Sixth graders	.07 (.09)	.03 (.04)	.02 (.04)	.01 (.04)	.00 (.02)
RT (ms)					
Kindergarteners	2,336.89 (958.97)	2,055.08 (719.92)	1,996.23 (706.84)	1,945.75 (763.50)	1,717.96 (550.80)
First graders	1,507.64 (378.24)	1,333.75 (344.42)	1,260.13 (316.34)	1,181.49 (262.00)	1,159.77 (280.90)
Second graders	1,072.47 (198.95)	989.20 (187.99)	923.92 (161.74)	868.58 (132.72)	866.11 (174.23)
Sixth graders	700.00 (112.09)	625.08 (105.61)	599.02 (74.89)	584.18 (77.48)	573.38 (76.83)

while saying: 'This line goes from 0 (dots) to 10 (or 100) (dots). If here is 0 and here is 10 (or 100), where would position this number (quantity)?'. After that, the children were able to go through all sheets at their own pace. Kindergarteners only solved a 0–10 number line task. First graders performed both a 0–10 and a 0–100 tasks. Second and sixth graders performed only the 0–100 task.

Results

First, the results of the number knowledge task are discussed after which we present the results of the number comparison and the number line estimation tasks. After that we describe the correlational analyses and their development across grades.

Number knowledge task

All children were able to read aloud the digits correctly. Two kindergartners scored 6/9 (errors for the digits 6, 7, and 8) and one kindergartner scored 7/9 (errors for the digits 7 and 8). The remaining kindergartners and all first graders scored 9/9. All children were included in the analyses.

Number comparison

RTs were adjusted to reflect both speed and accuracy using the formula RT/(1 – error rate). This was done to control for potential speed-accuracy tradeoffs (see Simon *et al*., 2008 for a similar method).

The mean error rates[1] and adjusted RTs are shown by distance and grade in Table 1 (symbolic condition) and Table 2 (non-symbolic condition). The effect of distance was examined with repeated measures analysis of variance with distance as within-subject factor and grade as between-subjects factor on children's adjusted RTs.

[1] *Tables 1 and 2 also depict mean error rates on the symbolic and non-symbolic comparison tasks, which allow one to calculate the raw reaction times from the adjusted reaction times for each distance, task, and grade.*

Table 2. Mean error rates, mean adjusted reaction times (RTs), and corresponding (standard deviations) for the five distances (d1–d5) of the non-symbolic condition of the number comparison task, per grade

	d1	d2	d3	d4	d5
Error rates (%)					
Kindergarteners	.13 (.12)	.08 (.08)	.03 (.05)	.03 (.07)	.01 (.04)
First graders	.11 (.09)	.05 (.07)	.01 (.04)	.01 (.03)	.02 (.04)
Second graders	.12 (.08)	.07 (.07)	.02 (.05)	.00 (.02)	.00 (.02)
Sixth graders	.10 (.07)	.04 (.05)	.01 (.03)	.01 (.03)	.01 (.04)
RT (ms)					
Kindergarteners	2,063.92 (449.26)	1,649.33 (413.45)	1,494.38 (366.09)	1,406.60 (329.51)	1,323.92 (335.80)
First graders	1,840.65 (518.62)	1,387.44 (323.27)	1,285.64 (366.73)	1,172.06 (327.80)	1,160.06 (341.83)
Second graders	1,354.40 (289.84)	1,080.32 (191.04)	961.14 (218.85)	931.41 (192.74)	856.39 (164.28)
Sixth graders	900.90 (124.61)	749.30 (139.45)	679.21 (110.49)	643.37 (122.03)	615.98 (93.28)

Symbolic condition

There was a main effect of distance ($F(4,111) = 21.37$, $p < .001$, $\eta_p^2 = .44$). There was also a main effect of grade ($F(3,114) = 85.90$, $p < .001$, $\eta_p^2 = .69$), indicating that the RTs decreased with increasing grade. There was also a distance $\times$ grade interaction ($F(12,294) = 2.41$, $p < .01$, $\eta_p^2 = .08$), showing group differences between the youngest and the oldest children were most prominent at smaller distances (Table 1).

Non-symbolic condition

The main effect of distance on the RTs ($F(4,111) = 128.44$, $p < .001$, $\eta_p^2 = .82$) was found. There was a main effect of grade ($F(3,114) = 65.76$, $p < .001$, $\eta_p^2 = .63$), indicating that RTs decreased with increasing grade. There was distance $\times$ grade interaction ($F(12,294) = 4.54$, $p < .001$, $\eta_p^2 = .14$), showing that the difference in RTs between the youngest and the oldest children was the largest at the smallest distances (Table 2).

To examine the CDE in more detail, we computed the size of the distance effect for each child by calculating the slope of a regression in which distance predicted the adjusted RTs. The size of the slope reflects the distance effect, with steeper slopes indicating larger distance effects. Table 3 shows the mean slope of the adjusted RTs per grade for each condition. As expected, these mean slopes were negative, reflecting the negative relationship between distance and adjusted RT. One sample *t*-tests revealed that the slopes were significantly different from zero for all grades (ts > −3.64, ps < .01). The size of this slope decreased with increasing grade (symbolic: $F(3,114) = 6.51$, $p < .001$; non-symbolic: $F(3,114) = 17.18$, $p < .001$).

Number line estimation

The percentage absolute error (PAE) was calculated per child as a measure of children's estimation accuracy. This was done by using the following formula by Siegler and Booth (2004):

$$\left|\frac{Estimate - EstimatedQuantity}{Scale\ of\ Estimates}\right|$$

.

Table 3. Mean slope and standard deviation of the adjusted reaction times of the number comparison task and mean percent absolute error (PAE) and the corresponding standard deviations of the number lines, per grade, for both the symbolic and non-symbolic condition

	Number comparison task		Number line estimation task			
	M_{slope}	SD_{slope}	$M_{PAE0-10}$	$SD_{PAE0-10}$	$M_{PAE0-100}$	$SD_{PAE0-100}$
Symbolic condition						
Kindergarteners	−134.72	188.75	.17	.05		
First graders	−84.80	56.83	.20	.09	.12	.05
Second graders	−53.34	34.78			.08	.04
Sixth graders	−29.41	22.18			.03	.01
Non-symbolic condition						
Kindergarteners	−172.27	73.63	.17	.07		
First graders	−157.65	86.48	.17	.10	.19	.06
Second graders	−116.65	52.39			.14	.07
Sixth graders	−67.58	21.64			.09	.04

For example, if a child was asked to estimate 18 on a 0–100 number line and placed the mark at the point on the line corresponding to 30, the PAE would be (30–18)/100 or 12%.

Table 3 shows the mean PAE per grade, number lines, and conditions. There were no differences between kindergarteners and the first graders on the 0–10 number lines (symbolic: $F(1,54) = 1.66$, $p = .20$, $\eta_p^2 = .03$; non-symbolic: $F < 1$). Accuracy on the symbolic ($F(2,90) = 48.96$, $p < .001$, $\eta_p^2 = .52$) and non-symbolic ($F(2,90) = 27.53$, $p < .001$, $\eta_p^2 = .38$) 0–100 tasks increased with grade.

We further analyzed children's estimation patterns by fitting linear and logarithmic functions to the group means and to each individual child (Siegler & Opfer, 2003). For group means, a paired *t*-test was conducted on the mean R^2 linear and mean R^2 logarithmic for each group. If the *t*-test indicated a significant difference between both *R*-squares, the best fitting model (linear or logarithmic) was attributed to the group.

Symbolic condition

For the 0–10 interval, the model with the highest R^2 was linear for both the kindergarteners ($R^2_{lin} = .63$) and the first graders ($R^2_{lin} = .80$) and differed significantly from the logarithmic fit in both grades (R^2_{log} kindergarten $= .47$; $t(25) = 6.12$, $p < .001$ and R^2_{log} first grade $= .65$; $t(29) = 12.81$, $p < .001$). For the 0–100 interval, the fit of the logarithmic model for the first graders ($R^2_{log} = .81$) did not differ from the linear fit ($R^2_{lin} = .76$; $t(29) = -1.71$, $p = .10$). For the second and the sixth graders, the fit of the linear model was the best ($R^2_{lin} = .91$ and $R^2_{lin} = .99$, respectively) and differed significantly from the logarithmic fit (R^2_{log} second grade $= .87$; $t(28) = 2.56$, $p = .02$ and R^2_{log} sixth grade $= .84$; $t(32) = 33.03$, $p < .001$).

Non-symbolic condition

For the 0–10 interval, the model with the highest R^2 was linear for kindergarteners ($R^2_{lin} = .72$) and first graders ($R^2_{lin} = .83$) and differed significantly from the logarithmic model in both grades (kindergarten $R^2_{log} = .62$; $t(25) = 5.29$, $p < .001$ and first grade $R^2_{log} = .67$; $t(29) = 21.32$, $p < .001$). For the 0–100 interval, the logarithmic model provided a better fit than the linear model in first graders ($R^2_{log} = .88$; $R^2_{lin} = .76$;

Table 4. Partial correlations between the experimental tasks and between the experimental tasks and the standardized mathematics achievementscores, controlled for grade

	1	2	3	4	5	6
1. Standardized mathematics achievement						
2. Symbolic comparison slope	.22*					
3. Symbolic comparison adjusted reaction times	–.24**	–.49**				
4. Non-symbolic comparison slope	.08	.08	–.04			
5. Non-symbolic comparison adjusted reaction times	–.16	–.27**	.66**	–.25**		
6. Symbolic mean PAE	–.28**	–.12	.03	.02	.13	
7. Non-symbolic mean PAE	–.22*	.04	–.03	–.02	.18	.41**

*$p < .05$ (two-tailed).**$p < .01$ (two-tailed).

$t(29) = -5.45$, $p < .001$). In second graders, the linear model provided a better fit ($R^2_{\text{lin}} = .84$), however, not significantly different from the logarithmic fit ($R^2_{\text{log}} = .83$; $t < 1$). In sixth graders, the linear model fitted the best ($R^2_{\text{lin}} = .92$) and differed significantly from the logarithmic model ($R^2_{\text{log}} = .86$; $t(32) = 3.98$, $p < .001$).

Correlational analyses

Partial correlations controlling for grade were calculated to investigate the association between the experimental tasks and mathematics achievement level. The raw mathematical achievement scores were transformed to *z*-scores per grade. For number comparison, the adjusted RTs and the slope (i.e., the size of the distance effect) were used as indices. For the number line estimation, the mean PAE was employed as index of subjects' performance on the number lines. In first graders, the mean PAE of the 0–10 and 0–100 number line task was used. The results of these analyses are presented in Table 4.

We also investigated whether the association between the numerical processing tasks and mathematics achievement tests was specific to mathematics by calculating partial correlations, with spelling performance being additionally controlled for. Again, the raw scores of the children on this spelling test were transformed to *z*-scores per grade.

Significant correlations between mathematics achievement and the symbolic number comparison were found. Higher math achievement was associated with a faster RTs and smaller slopes. These correlations remained significant when spelling ability was controlled for (RTs: $r(115) = -.25$, $p < .01$; slope: $r(115) = .18$, $p = .05$). There were no significant correlations between the non-symbolic number comparison and mathematics achievement.

There was a significant negative association between the mean PAE and mathematics achievement for both symbolic and non-symbolic number line tasks, indicating that fewer errors on number line estimation were associated with better math performance. This association remained after controlling for spelling performance (symbolic: $r(115) = -.27$, $p < .01$; non-symbolic: $r(115) = -.25$, $p < .01$). Because measures of symbolic and non-symbolic number line estimation were highly correlated ($r(115) = .41$, $p < .01$), we further investigated whether the association between math performance and symbolic number line estimation remained when non-symbolic number line estimation was additionally controlled for, and whether the association between math performance and non-symbolic number line estimation remained when symbolic number line estimation was controlled for. This analysis revealed that the association between the symbolic number line estimation and mathematics achievement remained ($r(114) = -.21$, $p = .02$), but that the association between non-symbolic number line estimation and math achievement disappeared ($r(114) = -.12$, $p = .20$.).

Evolution of the associations over grades

To investigate the evolution of the associations between the experimental tasks and mathematics achievement over grades, we calculated an analysis of covariance with grade as factor and mathematics achievement and the interaction between grade and mathematics achievement for each of the experimental tasks (see Table 4).

Only for symbolic number comparison, the grade × mathematics achievement interaction was significant (RTs: ($F(3,110) = 7.63$, $p < .01$, $\eta_p{}^2 = .17$; slope: $F(3,110) = 7.12$, $p < .01$, $\eta_p{}^2 = .16$). This indicates that only for the symbolic number comparison task, the association with mathematics achievement changed significantly over grades. To examine this interaction in more detail, Pearson correlation coefficients were calculated per grade. For the RTs, associations were $r(24) = -.54$, $p < .01$ for kindergarteners, $r(28) = -.07$, $p = .70$ for first graders, $r(27) = -.08$, $p = .69$ for second graders, and $r(31) = -.17$, $p = .36$ for sixth graders, indicating that the association is only significant in kindergarteners. Turning to the associations with the number comparison slopes, significant associations with math achievement were observed for all ages (kindergarten: $r(24) = -.52$, $p < .01$; first grade: $r(28) = -.43$, $p = .02$; second grade: $r(27) = -.37$, $p = .05$; and sixth grade: $r(31) = -.39$, $p = .03$), with the largest association being observed in kindergarteners.

Discussion

The ability to represent number has been considered to be a key precursor of children's mathematical development (e.g., De Smedt, Verschaffel *et al.*, 2009). Two experimental tasks have been commonly used to investigate number processing and its association with mathematics achievement: number comparison and number line estimation. The majority of the existing studies mainly have used only one numerical notation (symbolic or non-symbolic) or one task, thereby largely focusing on one age group. The present study tried to extend these data by investigating a number comparison task and number line estimation task in both symbolic and non-symbolic notations and their associations with mathematical achievement in kindergarteners, first-, second, and sixth graders. Children's performance on the experimental tasks was consistent with previous findings. In the comparison tasks, we observed a decreasing distance effect with increasing age, which reflects an increase in precision of number representations when schooling advances (Holloway & Ansari, 2009; Sekuler & Mierkiewicz, 1977). In number line estimation, older children relied more on linear representations that resulted in fewer errors in putting a number on the number line, compared to younger children (Booth & Siegler, 2008; Ramani & Siegler, 2008).

A specific aim of the current study was to investigate the associations between the number processing tasks and mathematics achievement. Consistent with previous studies (De Smedt *et al.*, 2009; De Smedt & Gilmore, 2011; Holloway & Ansari, 2009), we observed an association between symbolic number comparison and mathematics achievement. This association was particularly strong in kindergarteners, but was reduced in older children, a finding that is consistent with data by Holloway and Ansari (2009). In kindergarten, children are in the process of mapping symbolic digits onto pre-existing non-symbolic representations (Barth *et al.*, 2005; Mundy & Gilmore, 2009), which might explain why symbolic number comparison is the most sensitive to individual differences in math achievement at this age. Hence, the present results suggest the importance of informal experiences with Arabic digits at home or in preschool, focusing on learning to connect symbols with their numerical meaning (e.g., exposure to symbolic numbers, books, or numerical board games, such as Chutes and Ladders).

There were no significant associations between mathematics achievement and non-symbolic number comparison, a finding that is consistent with most (De Smedt & Gilmore, 2011; Holloway & Ansari, 2009; Landerl & Kolle, 2009; Rouselle & Noël, 2007; Soltész *et al.*, 2010) but not all (Halberda *et al.*, 2008) existing data. This all indicates that the ANS does not contribute to individual differences in mathematics achievement. Importantly, however, it should be noted that our study used smaller numbers than Halberda *et al.* (2008), who also calculated the Weber fraction as a measure of numerical processing. This might explain the difference between our results and those obtained by Halberda *et al.* (2008), since it has already been demonstrated that different measures of the ANS do not necessarily provide similar results (Gilmore, Attridge, & Inglis, in press).

Turning to the number line estimation tasks, we observed, in line with Siegler and colleagues (Siegler & Booth, 2008; Ramani & Siegler, 2008), that more linear estimation patterns on the symbolic number line estimation task were associated with higher mathematics achievement. We extended these findings by additionally showing that more linear estimation patterns on a non-symbolic version of the number line task were associated with better mathematical performance. These associations between number line estimation and mathematics achievement were similar across the different age groups, yet the current sample sizes per age group were small, which might have obscured possible between-grade differences.

It is important to note that performance on both conditions of the number line estimation tasks were highly correlated. Partial correlations indicated that the association between mathematics achievement and symbolic task performance remained when non-symbolic performance was controlled for, whereas the association between mathematics achievement and non-symbolic task performance disappeared when symbolic performance was controlled for. This again indicates that symbolic rather than non-symbolic numerical representations are important for individual differences in mathematical development.

To summarize, the current study demonstrated that particularly symbolic measures of number representation are associated with individual differences in mathematics achievement. This all indicates that the access to the numerical meaning from symbolic digits rather than the representation of number *per se* is related to individual differences in mathematical development. The present study remained, however, cross-sectional and cannot determine whether symbolic number representations are precursors rather than a consequence of individual differences in mathematics achievement. Although longitudinal data by De Smedt *et al.* (2009) indeed showed that symbolic number representations predicted mathematics achievement 1 year later, future longitudinal data across larger age ranges are needed to further shed light on these issues.

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